HPRF Physics

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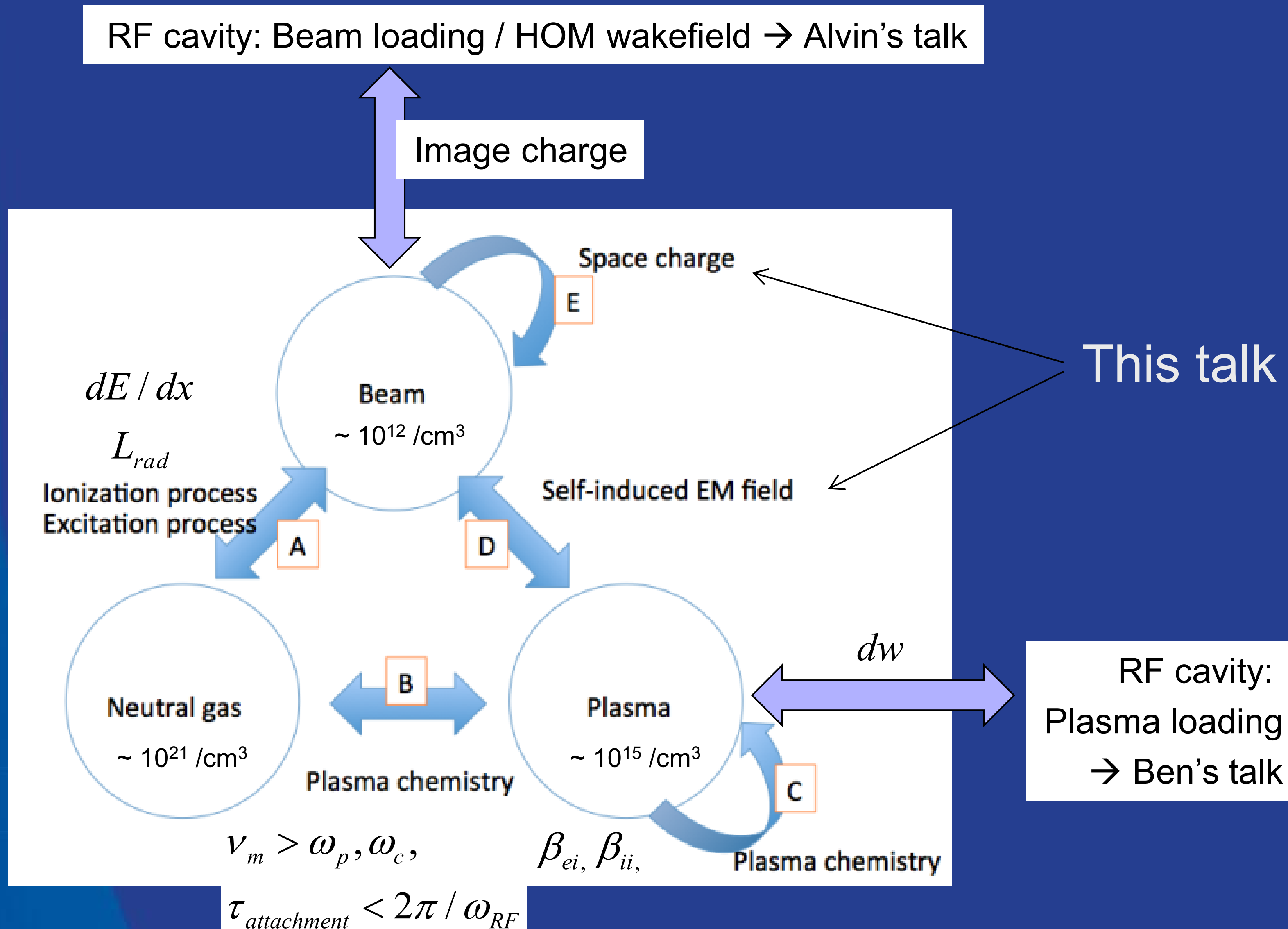
Nov. 7, 2013

HCC workshop

U.S. DEPARTMENT OF ENERGY

Fermilab



[Courtesy of K. Yonehara]

Outstanding Questions

1. Whether space-charge effects matter in HCC
   - Transverse
   - Longitudinal

2. How the muon beam interacts with EM field of plasma ?

   → Charge neutralization :

   (e.g., Final focusing of collider/HIF)

   → Any local charge buildup ?

   (e.g., PWFA)

PART 1:
Space Charge

Transverse Space-Charge

Equivalent uniform KV beam model:

$$\frac{d^2 r_b}{ds^2} + \left(\kappa_{sf} - \frac{K_b}{r_b^2}\right) r_b = \frac{\varepsilon_{KV}^2}{r_b^3}$$

$$r_b = \sqrt{2} R_b = \sqrt{2}\sqrt{< r^2 >} = 2\sqrt{< x^2 >}$$

$$\kappa_{sf} = \left(\frac{qB_s / \gamma m}{2\beta c}\right)^2$$

$$K_b = \frac{1}{4\pi\varepsilon_0 \gamma^2} \frac{2 N_L q^2}{\gamma m \beta^2 c^2} = \frac{1}{4\pi\varepsilon_0} \frac{2q^2}{\gamma^3 m \beta^2 c^2} \left(\frac{N}{\sqrt{2\pi}\sigma_z}\right)$$

$$\varepsilon_{KV} = 4\varepsilon$$

$$\varepsilon_{th}^2 = \sqrt{\det(\Sigma)} = \varepsilon_1 \varepsilon_2 = \varepsilon^2 - \frac{1}{4} L^2$$

$$\varepsilon = \varepsilon_{\perp,n} / (\beta\gamma)$$

$$\text{Figure of merit} = \frac{K_b}{r_b^2 \kappa_{sf,0}} \sim \frac{K_b}{\varepsilon_{KV}\sqrt{\kappa_{sf,0}}} \propto \frac{1}{\varepsilon_{\perp,n}\, \varepsilon_{\parallel,n}^{1/2}}$$

Longitudinal Space-Charge

Parabolic line charge density profile (Neuffer distribution):

$$\frac{d^2 z_m}{ds^2} + \left(\kappa_z - \frac{K_L}{z_m^3} \right) z_m = \frac{\varepsilon_{zz'}^2}{z_m^3}$$

$$z_m = \sqrt{5}\sqrt{< z^2 >}$$

$$\kappa_z = \frac{2\pi}{\lambda} \frac{qE_m(-\eta)\cos\varphi}{\beta^3 \gamma mc^2}$$

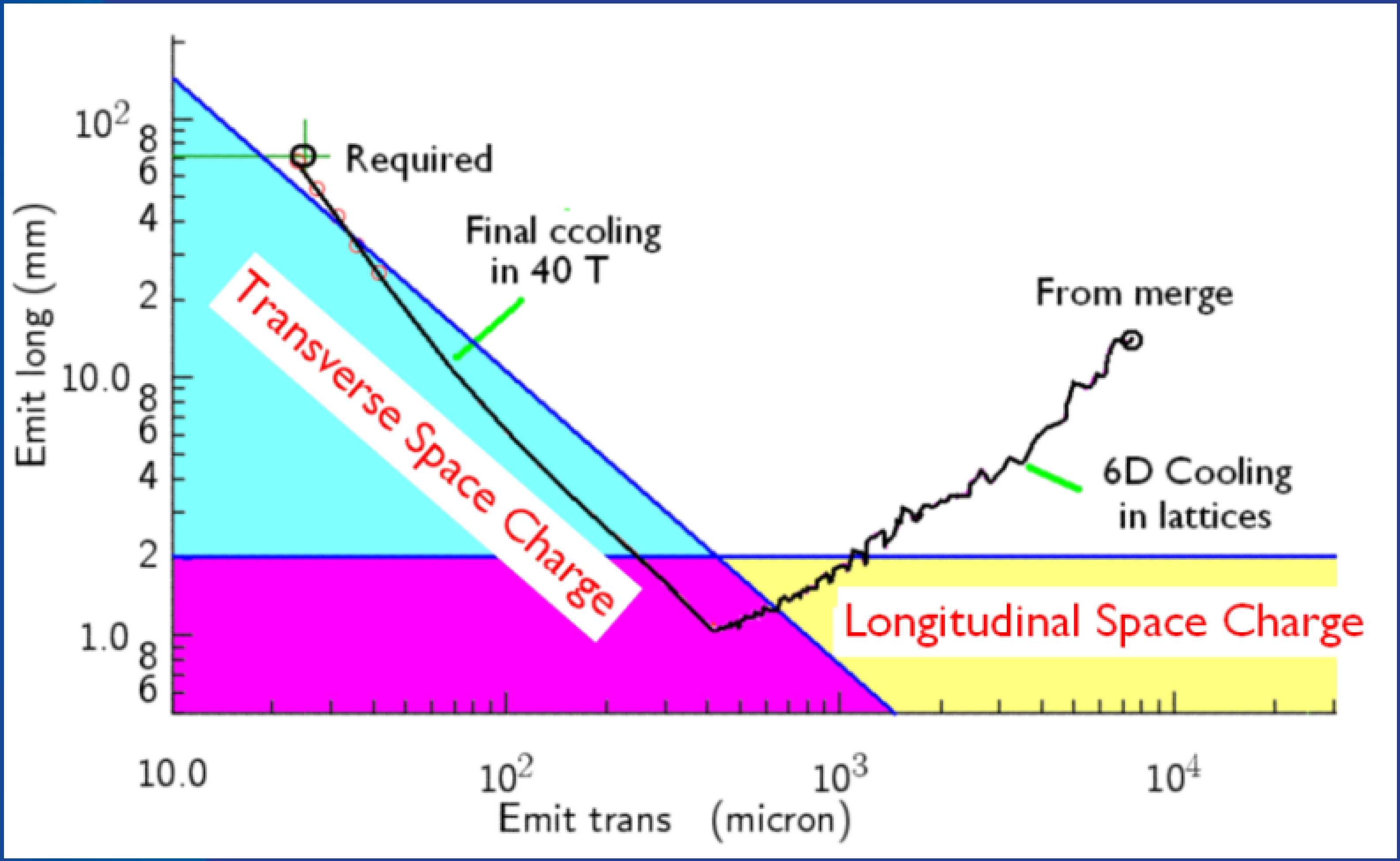
$$K_L = \frac{3}{2} \frac{gNr_c / \gamma}{\beta^2 \gamma^2} (-\eta)$$

below transition: $\eta < 0,\ 0 < \varphi < 90°$

above transition: $\eta > 0,\ 90° < \varphi < 180°$

$$\varepsilon_{zz'} = \frac{5\varepsilon_{\parallel,n} |\eta|}{\beta\gamma}$$

$$\text{Figure of merit} = \frac{K_L}{z_m^3 \kappa_{z0}} \sim \frac{K_L}{\varepsilon_{zz'}^{3/2} \kappa_{z0}^{1/4}} \propto \frac{1}{\varepsilon_{\parallel,n}^{3/2}}$$

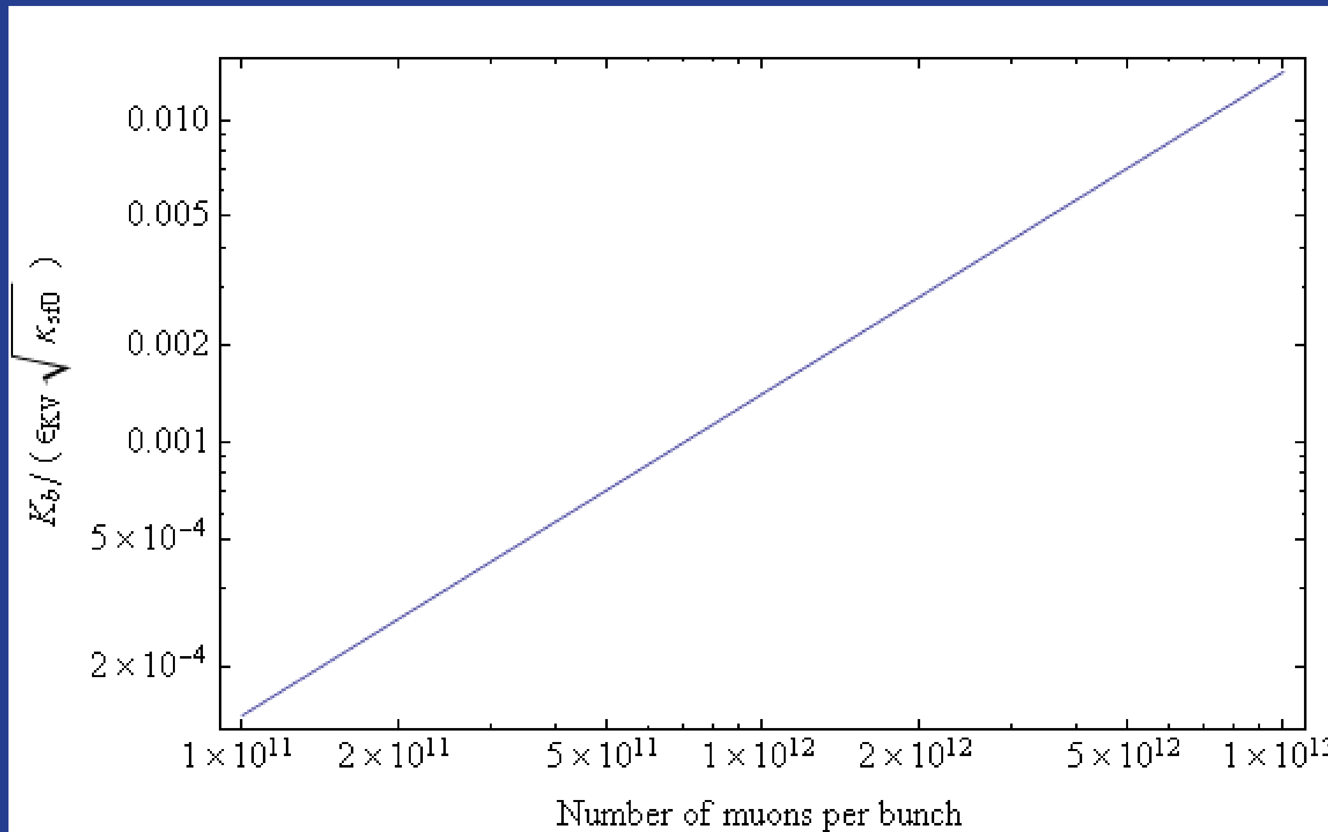


[Courtesy of R. Palmer]

$\varepsilon_{\perp,n} = 300\ \mu\text{m}$

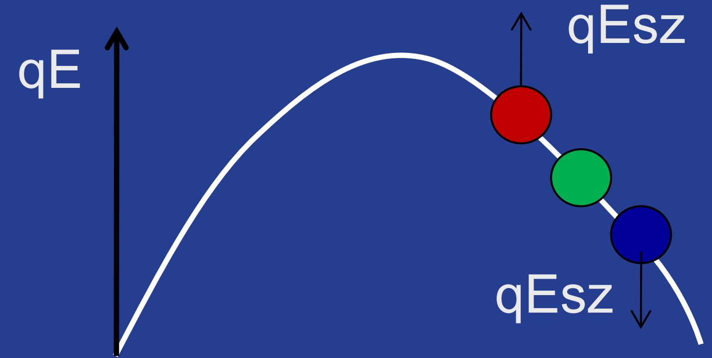
$R_b = 2\ \text{mm}$

$\sigma_z = 2\ \text{cm}$



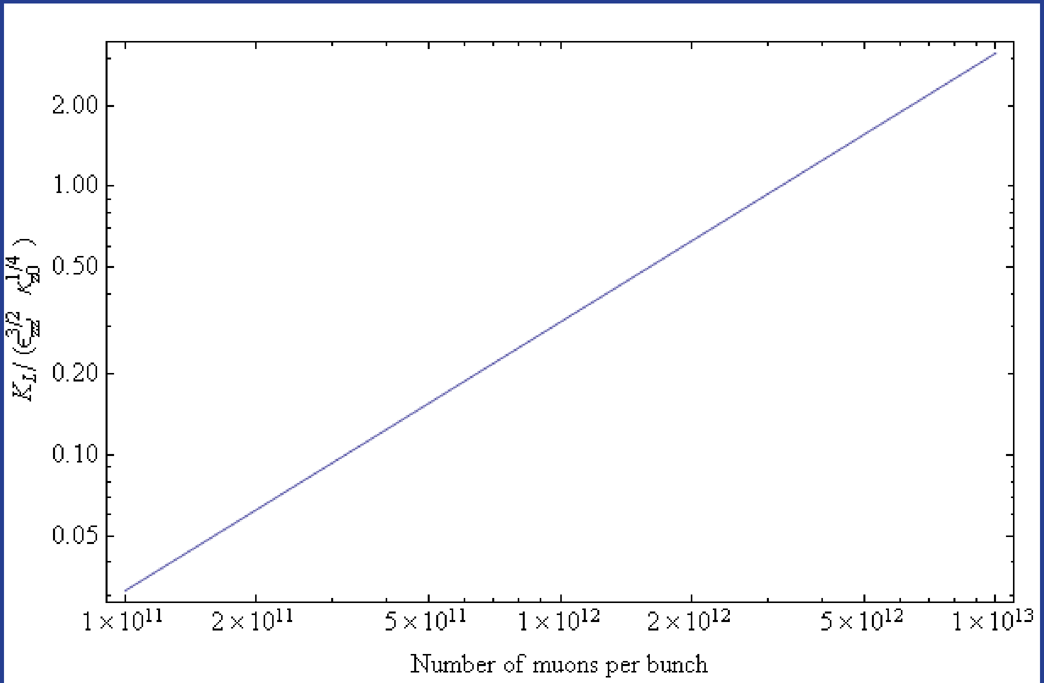
Transverse space charge is negligible at least up to the end of 6D HCC channel

For HCC: above transition → longitudinal space charge is indeed focusing



For linear channel → longitudinal space charge is defocusing

$$\varepsilon_{\|,n} = 1 \text{ mm}$$
$$\sigma_z = 2 \text{ cm}$$
$$\eta = -1/\gamma^2$$
$$g \sim 4$$



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PART 2:
Beam-Plasma Interactions

Plasma lens effect:

$$\frac{d^2 r_b}{ds^2} + \left(\kappa_{sf} + \frac{\alpha K_b}{r_b^2} \right) r_b = \frac{\varepsilon_{KV}^2}{r_b^3}$$

$$\alpha = \frac{[\beta^2(1-f_m)-(1-f_q)]}{1-\beta^2} = \begin{cases} -1 & \text{for } f_m = f_q = 0 \\ 0 & \text{for } f_m = 0, f_q = 1/\gamma^2 \\ \gamma^2\beta^2 & \text{for } f_m = 0, f_q = 1 \end{cases}$$

$$\frac{d^2 z_m}{ds^2} + (\kappa_z + 0) z_m = \frac{\varepsilon_{zz'}^2}{z_m^3}$$

Transverse: Since the space charge effect is already negligible, no additional benefit of plasma lens effect $\beta\gamma \sim 2$

Longitudinal: Plasma lens effect could eliminate longitudinal space charge force, but we are already neglecting it in most of HCC simulations

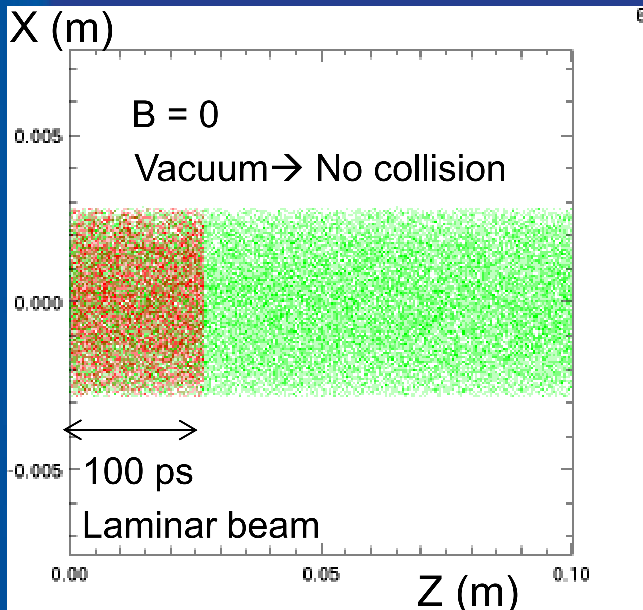
Simulation Goals

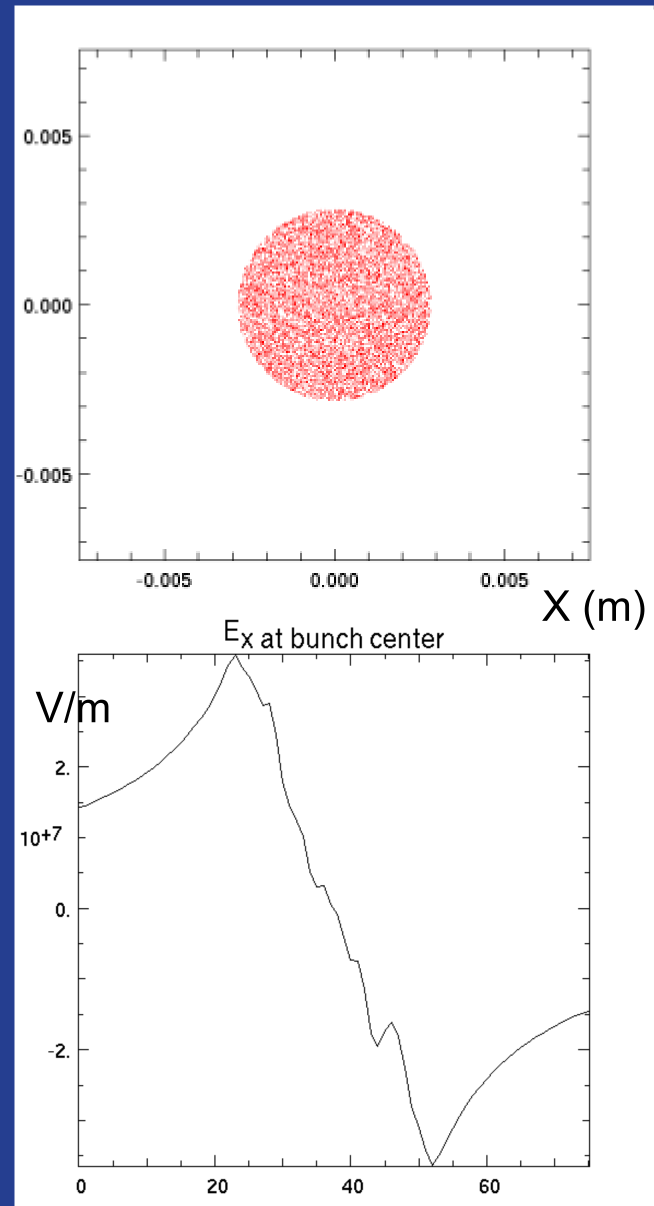
- To check our 0^{th} order analytical model (no plasma wake/oscillation, charge neutralization)
- To verify that there are no other effects of beam-plasma interactions we're missing (i.e., the plasma is quiescent ?)

Simulation Efforts

- BNL (R. Samulyak / K. Yu)
  SPACE code: PIC + Fluid, beam-induced plasma
- Fermilab (M. Chung / A. Tollestrup / B. Freemire / K. Yonehara)
  WARP code: PIC with preformed plasma

Case 1: Beam density ~ Plasma density





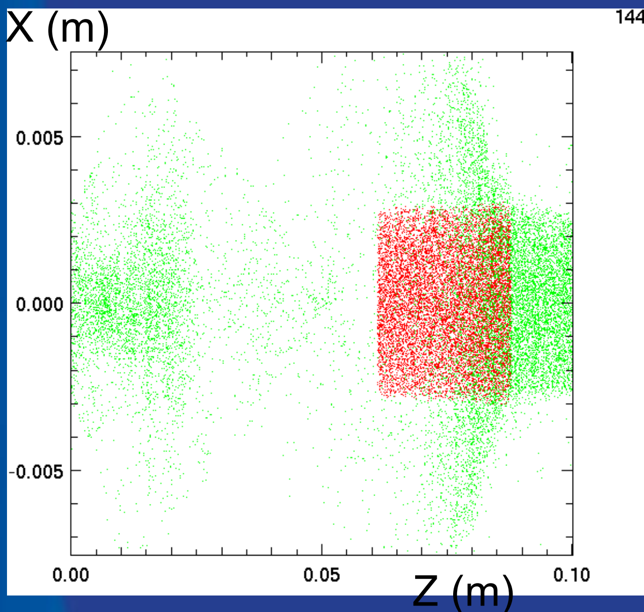
mu- : red (10,000 particles)

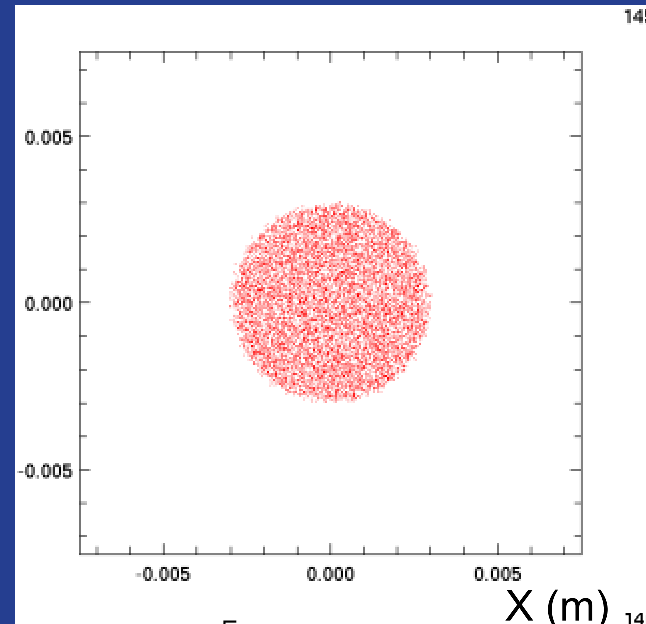
Electron: green

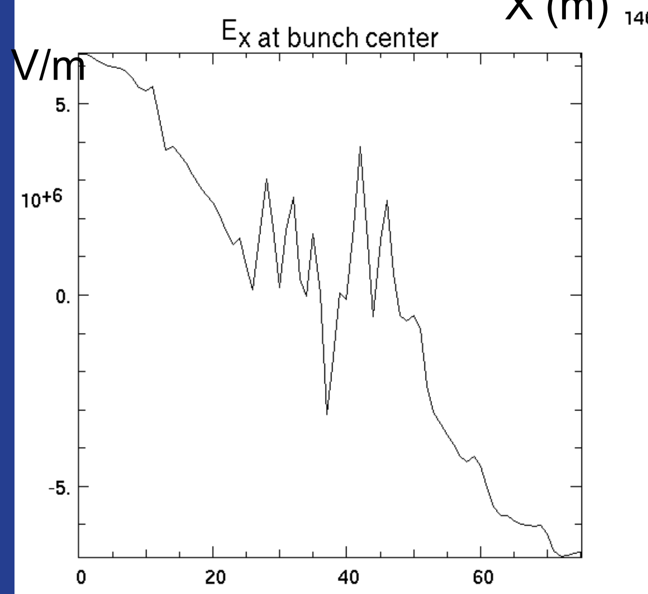
Positive ion: not shown

→ Same initial distribution as electrons

Case 1: Beam density ~ Plasma density (continued)







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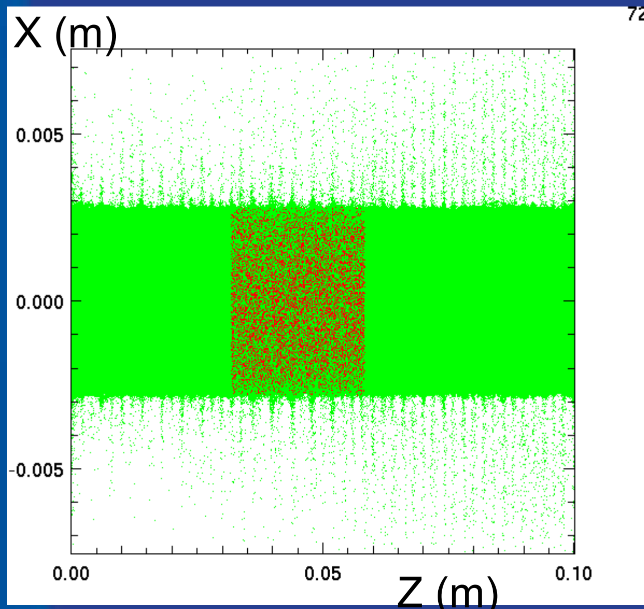
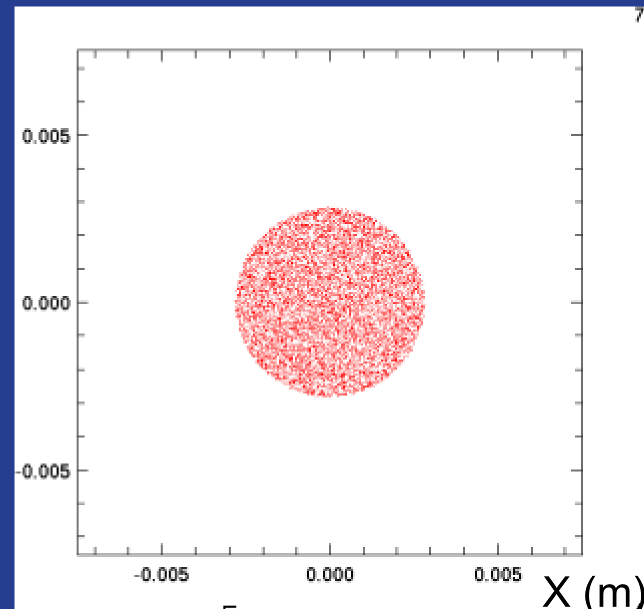
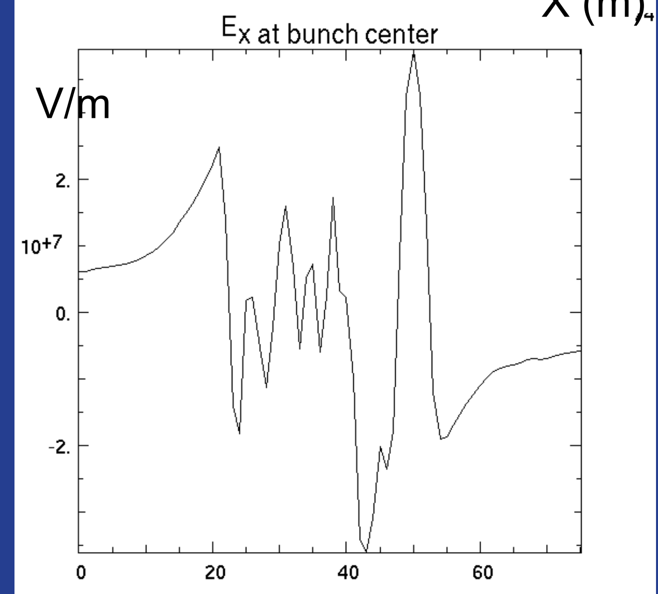
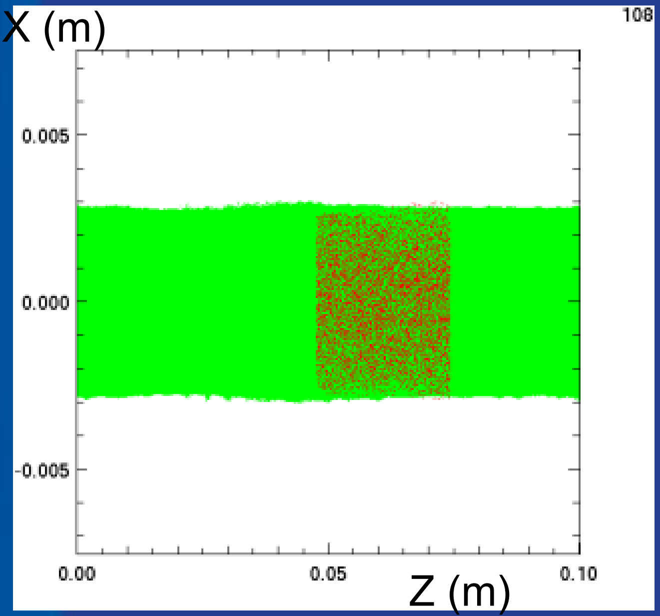
Case 2: Beam density ~ 100 x Plasma density

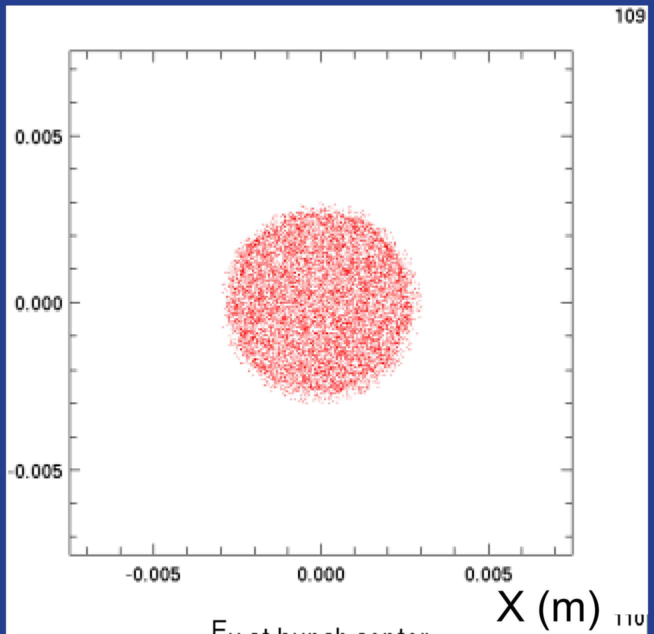


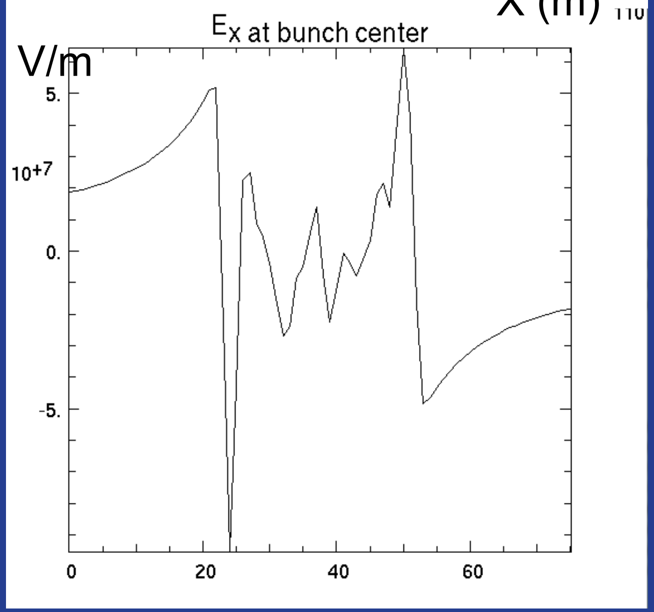




Case 3: Beam density ~ 100 x Plasma density, electron mass x100







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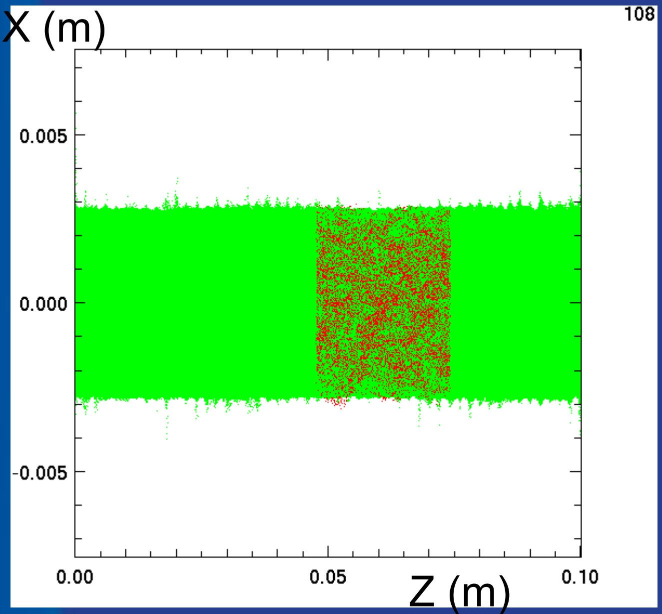
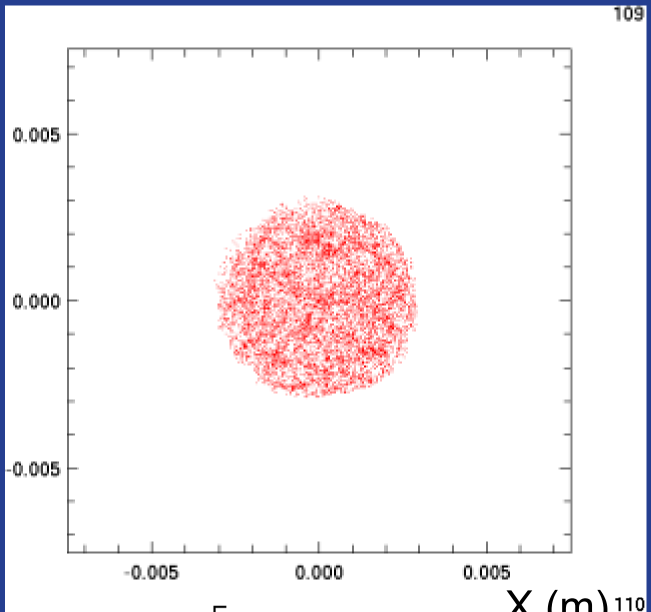
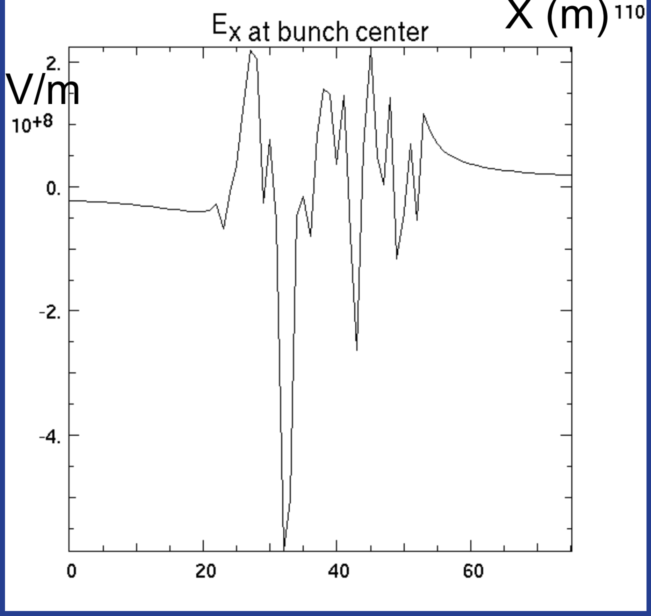
Case 4: Beam density ~ 100 x Plasma density, weight = 10
electron mass x100







Conclusions (or wish lists)

1. Space-charge effects seem to be manageable in HCC.

2. As the collision frequency is so high, the plasma seems to be quiescent.

3. As the plasma density is much higher than the beam density, full charge neutralization could occur. This would not change the present HCC performance, but could be useful beyond HCC.